Assignment - V

1. In a material for which $\sigma = 5(\Omega\text{-m})^{-1}$ and electric field intensity is $\vec{E} = 250 \sin 10^{10} \hat{a}_z$ V/m. Find conduction current and displacement current

Sol Given

$$\vec{E} = 250 \sin 10^{10} \hat{a}_z$$

conduction current $J = \sigma \vec{E}$

$$\vec{J} = 5(250 \sin 10^{10} \hat{a}_z) \text{ V/m}$$

$$\boxed{\vec{J} = 1250 \sin 10^{10} \hat{a}_z \text{ A/m}^2}$$

Displacement current is given by

$$J_D = \frac{\partial D}{\partial t}$$

$$= \epsilon \frac{\partial E}{\partial t}$$

$$= \epsilon_0 \epsilon_r \frac{\partial E}{\partial t}$$

$$= 8.85 \times 10^{-12} \times 1 \times \frac{\partial}{\partial t}(250 \sin 10^{10} \bar{a}_z)$$

$$J_D = 8.85 \times 10^{-12} \times 250 \cos 10^{10} \bar{a}_z$$

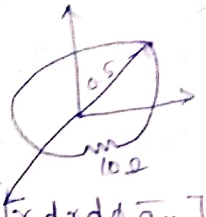
$$\boxed{J_D = 2212.5 \cos 10^{10} \bar{a}_z \text{ A/m}}$$

2. A circular loop conductor lies in $z=0$ plane and has a radius of 0.5m, resistance of 10Ω and $\vec{B} = 1.9 \sin 10^3 t\ \hat{a}_z$. Determine the current in the loop.

Sol:- Given that $B = 1.9 \sin 10^3 t\ \bar{a}_z$

$r = 0.5$m

Resistance $= 10\,\Omega$



$d\bar{s} = (r\,dr\,d\phi)\,\bar{a}_z$

Total flux is $\phi = \int_s \bar{B} \cdot d\bar{s}$

$$= \int_{\phi=0}^{2\pi} \int_{r=0}^{0.5} \left[(1.9 \sin 10^3 t)\,\bar{a}_z\right]\left[r\,dr\,d\phi\,\bar{a}_z\right]$$

$$= (1.9 \sin 10^3 t)\left[\phi\right]_0^{2\pi}\left[\frac{r^2}{2\pi}\right]_0^{0.5}$$

$$= (1.9 \sin 10^3 t)\left[2\pi\right]\left[\frac{(0.5)^2}{2\pi}\right]$$

$$= (1.9 \sin 10^3 t)\, 2\pi \left[\left(\frac{(0.5)^2}{2\pi}\right)\right]$$

$$= 1492.25 \sin 10^3 t \text{ mwb.}$$

Now induced emf

$$e = \frac{-d\phi}{dt}$$

$$= \frac{-d}{dt}\left[1492.25 \times 10^{-3} \sin 10^3 t\right]$$

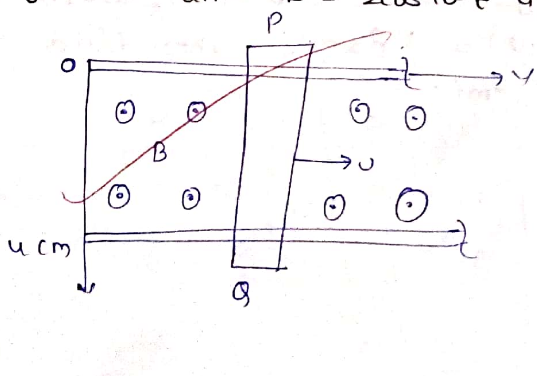
$$= -1.4922 \cos 10^3 t \text{ V}$$

$$i = \frac{e}{R} = \frac{-1.4922 \cos 10^3 t}{10}$$

$$= -0.14922 \cos 10^3 t$$

3. Length of bar is 4cm and placed along x-axis and it can ~~be~~ slide freely over two conducting rails along y-axis. Calculate the induced emf if the bar is stationed at $y = 6$cm and $\vec{B} = 2\cos 10^5 t\ \hat{a}_z$ wb/m²

Sol



Given $\vec{B} = 2\cos 10^5 t\, \hat{a}_z$.

$$\frac{\partial \vec{B}}{\partial t} = 2(-\sin 10^5 t) \times 10^5\, \hat{a}_z$$

(a) induced emf

$$V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t}\, ds$$

$$= \int_{x=0}^{4\times10^{-2}} \int_{y=0}^{6\times10^{-2}} -2\times10^5 \times \sin 10^5 t\, \hat{a}_z$$

$$= -2\times10^5\times \sin 10^5 t \int_{x=0}^{0.04}\int_{y=0}^{0.06} dy\, dx$$

$$= -2\times10^5 \times \sin 10^5 t \int_{x=0}^{0.04} [y]_0^{0.06}\, dx$$

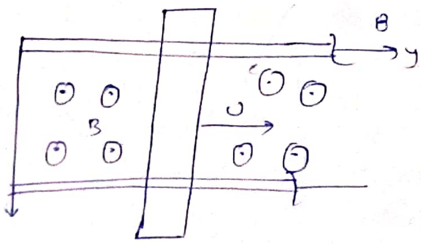
$$= -2\times10^5 \times 0.06 \times \sin 10^5 t \int_{x=0}^{0.04} dx$$

$$= -2\times10^5 \times 0.06 \times \sin 10^5 t \times 0.04$$

$$\boxed{V_{emf} = -480 \sin 10^5 t \ \text{V}}$$

4. A rod with length 8 cm is moving along y-axis. If $\vec{B} = 2\hat{a}_z$ wb/m² and velocity (V) = $\frac{1}{2}\hat{a}_x$ m/s then find induced emf.

Sol



Given $\vec{B} = 2\hat{a}_z$ wb/m²

$U = \frac{1}{2}\hat{a}_x$ m/sec

$$V_{emf} = \int_L (U \times B)\, dl$$

$$= \int_{x=l}^{0} (U\bar{a}_x \times B\bar{a}_z)\, dy\, \bar{a}_y$$

$$= \int_{x=8}^{0} \frac{1}{\cancel{2}} \times \cancel{2}\, dy\, \bar{a}_y$$

$$= \int_{x=8}^{0} dy\, \bar{a}_y$$

$$= [0-8]$$

$$\boxed{V_{emf} = -8\ \text{V}}$$

5. A parallel plate capacitor with plate area $3\,cm^2$ and plate separation of 3mm has voltage (V) = $10\sin 10^3 t$ volts applied to its plates. Calculate the displacement current. $\epsilon = 3\epsilon_0$

Sol

$$D = \epsilon E = \epsilon \frac{V}{d}.$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon}{d}\frac{dV}{dt}$$

Hence

$$I_d = J_d \cdot S = \frac{\epsilon S}{d}\frac{dV}{dt} = C\cdot\frac{dV}{dt}$$

which is the same as conduction current, given by

$$I_C = \frac{dQ}{dt} = \frac{S \cdot d\rho_s}{dt} = S \cdot \frac{dD}{dt}$$

$$= \epsilon S \frac{dE}{dt} = \frac{\epsilon S}{d} \cdot \frac{dV}{dt} \qquad \left[\because C = \frac{\epsilon_0 \epsilon_r A}{t}\right.$$

$$= C \frac{dV}{dt}$$

$$I_d = \cancel{3} \cdot \frac{10^{-\cancel{9}}}{\cancel{36\pi}_{\ 12\ 6}} \times \frac{\cancel{5} \times 10^{-4}}{\cancel{3} \times 10^{-\cancel{3}}} \times 10^{\cancel{5}} \times \cos 10^3 t \ \cancel{\times 10^3}$$

$$= \frac{5}{6} \times 10^{-4}$$

$$= 0.833 \times 10^{-4}$$

$$I_d = 0.083\,\text{mA}$$